Exercise 3: Line loads and Dirac deltas

08.11.2024 - 11.11.2024

Question 1

Figures (A)–(D) show a beam of length a which is mounted on a wall at point A. A line load is imposed on the beam.

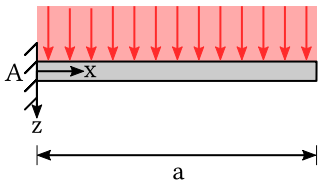
(a) Calculate the bending moment about the y-axis and the force in z-direction at point A!

(b) Each line load can be replaced by an equivalent point load, which generates the same moment and force at the support. Find the magnitude and line of action of this force!

Note: Introducing an equivalent force as in part (b) is useful for calculating reaction forces at supports. However, such a replacement must not be made when calculating *internal* forces!

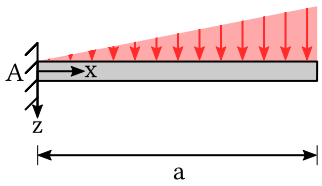
(A)

$q(x) = q_0$



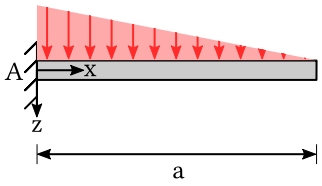
(B)

$q(x) = q_0 \frac{x}{a}$



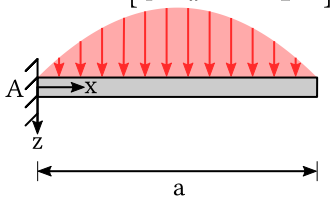
(C)

$q(x) = q_0 \left(1 - \frac{x}{a}\right)$



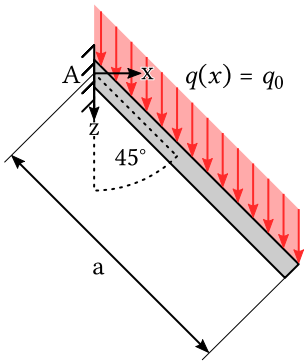
(D)

$q(x) = q_0 \left[\frac{1}{4} - \frac{1}{a^2}\left(x - \frac{a}{2}\right)^2\right]$



Question 2

A beam of length a is mounted on the wall at an angle of $45°$. A constant line load q_0 is applied. Calculate the reaction forces and the bending moment about the y-axis at point A!



Question 3

Recall that we discussed the Dirac Delta function $\delta(x)$. Evaluate the following definite integrals!

(a) $\int_{-10}^{10} \left(x^2 - 2x + 1\right) \delta(x - 2) dx$

(b) $\int_{-\infty}^{+\infty} \left(x^2 - 2x + 1\right) \delta(x + 10) dx$

(c) $\int_{-\infty}^{+\infty} \left(f(x) - f(x_0)\right) \delta(x - x_0) dx$

Question 4

A beam of length a is mounted on the wall. A force of magnitude F is applied in positive z-direction in the middle of the beam. Calculate the internal forces and moments *using integration*!