

## Exercise 7: Strain

06.12.2024 - 09.12.2024

**Question 1** .....

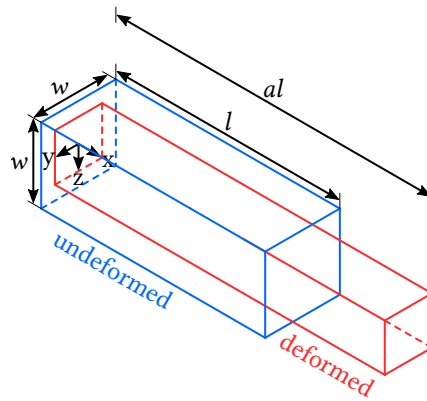
Consider the following displacement field,

$$\mathbf{u}(x, y, z) = k \begin{bmatrix} 2x + y^2 \\ x^2 - 3y^2 \\ 0 \end{bmatrix},$$

where  $k$  is a nonzero constant. Calculate the strain tensor  $\varepsilon$ !

**Question 2** .....

A solid bar with dimensions  $l \times w \times w$  (see below) is stretched along its length to a final length  $al$ . The volume of the bar does not change during deformation. Calculate the displacement vector  $\mathbf{u}$  and the strain tensor  $\varepsilon$ !



**Question 3** .....

(Saint-Venant's compatibility conditions)

The strain tensor  $\varepsilon$  has six distinct components. However, these six components are computed from only three components of the displacement vector  $\mathbf{u}$ . Thus, if we want to solve for the components of  $\mathbf{u}$  given the component of  $\varepsilon$ , we have six equations for three unknowns. For this system of equations to have a solution, some of the strain components must be related. Show that they are by considering their second derivatives! For example, differentiate  $\varepsilon_{xx}$  twice with respect to  $y$ ,  $\varepsilon_{yy}$  twice with respect to  $x$  and  $\varepsilon_{xy}$  with respect to  $x$  and  $y$ , and compare!

**Question 4** .....

In the first question, you computed the strain tensor  $\varepsilon$  for displacement field

$$\mathbf{u}(x, y, z) = k \begin{bmatrix} 2x + y^2 \\ x^2 - 3y^2 \\ 0 \end{bmatrix}.$$

Now show that  $\varepsilon$  fulfills the compatibility conditions!