Exercise 8: Hooke's law

13.12.2024 - 16.12.2024

Question 1

Reference: Chou, Pagano, *Elasticity: Tensor, Dyadic, and Engineering Approaches*, Dover Publications, p. 64.

Determine the slope of the σ_{xx} vs. ε_{xx} curve in the elastic range if a material is tested under the following state of stress:

$$\sigma_{xx} = 2\sigma_{yy} = 3\sigma_{zz}$$
$$\sigma_{xy} = \sigma_{xz} = \sigma_{yz} = 0$$

Question 2

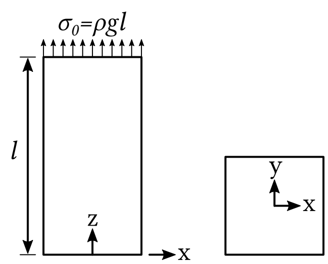
Reference: Chou, Pagano, *Elasticity: Tensor, Dyadic, and Engineering Approaches*, Dover Publications, p. 86.

A bar of constant mass density ρ hangs under its own weight and is supported by the uniform stress σ_0 as shown in the figure. Assume that the stresses σ_{xx}, σ_{yy}, σ_{xy}, σ_{xz}, and σ_{yz} vanish identically.

(a) Recall that there are 15 governing equations in $3D$: three equilibrium equations, six strain-displacement relations, and six stress-strain relations. Show that the 15 equations reduce to seven equations under the assumptions above. What are the variables?

(b) Integrate the equilibrium equation to show that $\sigma_{zz} = \rho g z$, where g is the acceleration due to gravity. Also show that the prescribed boundary conditions are satisfied by this solution.

(c) Find ε_{xx}, ε_{yy}, and ε_{zz} from Hooke's law.



Question 3

Metal or semiconductor crystals may contain defects in their lattice structure called "dislocations". These are very important for understanding plastic deformation. A so-called "screw dislocation", sketched in the figure, is created by the following displacement

$$\mathbf{u}(x,y,z) = \begin{bmatrix} 0 \\ 0 \\ \frac{b}{2\pi}\arctan\left(\frac{y}{x}\right) \end{bmatrix}.$$

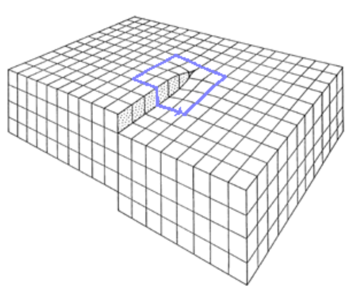


Figure 1: screw dislocation from: `https://www.tf.uni-kiel.de/matwis/amat/defen/kap5/backbone/r522.html`

Calculate the associated strain tensor ε and the stress tensor σ (using Hooke's law)! Is the body in a state of plane strain or plane stress? Do you notice something peculiar near the center of the dislocation at $x = y = 0$?

Question 4 . . . .

We now consider a state of plane strain. The governing equations are

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \quad \text{(definition of strain)},$$

$$\sigma_{xx} = 2\mu\varepsilon_{xx} + \lambda\left(\varepsilon_{xx} + \varepsilon_{yy}\right), \quad \sigma_{yy} = 2\mu\varepsilon_{yy} + \lambda\left(\varepsilon_{xx} + \varepsilon_{yy}\right), \quad \sigma_{xy} = 2\mu\varepsilon_{xy} \quad \text{(Hooke's law)},$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + F_x = 0, \quad \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} + F_y = 0, \quad \text{(equilibrium)}.$$

These are eight governing equations. However, we can combine them in such a way that we end up with only two equations in terms of the displacement components u_x and u_y. This form is convenient for problems where displacement components are prescribed over the entire boundary of the body. Find these two equations!