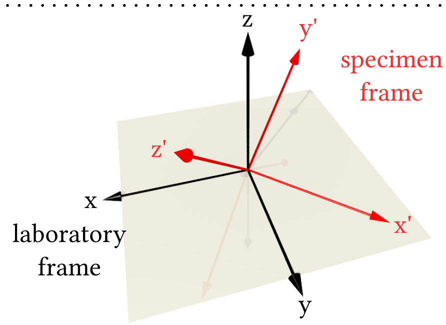
Exercise 10: Stress tensor

17.01.2025 - 20.01.2025

Question 1 . . . .

In this exercise, we will practice tensor rotation. Consider the two coordinate systems in the figure on the right. The red coordinate system ("specimen frame") has been rotated. The basis vectors of this system with respect to the laboratory frame are



$$\mathbf{x}' = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\\0\end{bmatrix}, \qquad \mathbf{y}' = \frac{1}{\sqrt{6}}\begin{bmatrix}1\\1\\-2\end{bmatrix}, \qquad \mathbf{z}' = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}.$$

Suppose we are given the representation of a stress tensor in the *specimen frame*,

$$\boldsymbol{\sigma}' = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & \sigma_{zz}\end{bmatrix}.$$

This stress tensor would be created by a force that acts on the plane whose normal is $\mathbf{z}'$, along the $\mathbf{z}'$-direction. We want the representation $\boldsymbol{\sigma}$ of this stress tensor in the *laboratory frame*.

(a) Find the rotation matrix $\mathbf{R}$ which, given the representation of a vector in the laboratory frame, yields the representation in the specimen frame upon matrix-vector multiplication!

(b) Verify that the determinant of $\mathbf{R}$ is equal to 1!

(c) Perfom tensor rotation to obtain $\boldsymbol{\sigma}$!

(d) Calculate the von Mises stress for $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}$!

Question 2 . . . .

Analyse the plane stress

$$\underline{\sigma} = \begin{pmatrix}3 & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & 2\end{pmatrix}$$

(a) Find the rotation angle $\phi_{\sigma,\max}$ at which the diagonal stress entries are maximal.

(b) Which values take the principal or main stresses σ_1 and σ_2?

(c) Find the rotation angle $\phi_{\tau,\max}$ at which the shear stress is maximal and compute the value for the maximal shear stress $\tau_{\max}$.

(d) In the lecture it was shown that not only the principal stresses can characterize a stress state but also the stress invariants. Compute the stress invariants I_1 and I_2.

(e) The dimension of a stress as well as the dimension of the principal stresses is force per area. What are the dimensions of the two stress invariants I_1 and I_2.

Question 3 . . . .

The following stress tensor characterises a special stress state

$$\underline{\sigma} = \begin{pmatrix}\sqrt{2} & \sqrt{2}\\ \sqrt{2} & -\sqrt{2}\end{pmatrix}$$

(a) Compute the angle $\phi_{\sigma,\max}$ at which the normal stresses takes its maximal value.

(b) Use the general rotation matrix and the computed angle $\phi_{\sigma,\max}$ to rotate the stress state in the coordinate system of maximal normal stress. What are the values for the principal stresses?

(c) What is the special name for the stress state found in (b)?

(d) Find the representation of the stress where the shear stress becomes maximal.

Question 4 . . . .
Now we have a more general three dimensional stress state given by

$$\underline{\sigma} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

(a) Compute the three principal stresses which are the eigenvalues of the stress tensor.

(b) What are the values of the three invariants I_1, I_2 and I_3 of the given stress state?

(c) Compute the hydrostatic stress σ_h.

(d) Compute the deviatoric stress s_{ij} which is also called stress deviator.

(e) Which values take the invariants J_1, J_2 and J_3 of the stress deviator.

(f) What is the value of the von Mises stress?

(g) What is special about J_2 and why is the von Mises stress derived from J_2?